$$\left(\frac{x}{a}-\frac{y}{b}\right)^2+\left(\frac{x}{a}+\frac{y}{b}+1\right)\left(\frac{x}{a}+\frac{y}{b}-1\right)=0, \text{ i. e., } \frac{x^2}{a^2}+\frac{y^2}{b^2}=\tfrac{1}{2}.$$

Solved similarly by J. Scheffer.

256. Proposed by F. P. MATZ, Ph. D., Sc. D., Reading, Pa.

The bisectors of the four angles of any quadrilateral intersect in four points, all of which lie on the circumference of the same circle.

Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.; FREDERIC R. HONEY, Ph. D., Trinity College, Hartford, Conn.; R. D. CARMICHAEL, Hartselle, Ala., and A. H. HOLMES, Brunswick, Me.

Denote the quadrilateral by $ABCD$, and the bisectors of the angles A, B, C, D by a, b, c, d, respectively. Suppose AB, DC intersect at O; a, d at X; and b, c at Y. We can easily show that, if B lies between A and O,

$$\angle AXD=\tfrac{1}{2}\pi+\tfrac{1}{2}\angle AOD;$$
$$\angle BYC=\tfrac{1}{2}\pi-\tfrac{1}{2}\angle AOD.$$

Hence, a pair of opposite angles of the quadrilateral whose sides are, consecutively, a, b, c, d, are supplementary, and it is therefore cyclic.

*** The problem admits of the following interesting extension: If a', b', c', d' are the bisectors, respectively, of the exterior angles A, B, C, D, the six quadrilaterals whose consecutive sides are, respectively, $(abcd)$, $(abc'd')$, $(bcd'a')$, $(cda'b')$, $(dab'c')$, $(a'b'c'd')$, are cyclic. It is necessary that no two sides of the given figure be parallel. GREENWOOD.

Also solved by F. P. Matz, J. Scheffer, G. B. M. Zerr, and W. J. Greenstreet. A. H. Holmes also solved No. 252.

CALCULUS.

194. Proposed by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

Show that the volume of the solid generated by the revolution of a segment of a circle, less than a semi-circle, about the diameter parallel to its chord, is equal to that of a sphere having a diameter equal to the chord; and hence that the volume is independent of the magnitude of the original circle, the length of the chord being known.

Solution by W. L. TRYON, Ithaca, N. Y.; R. D. CARMICHAEL, Hartselle, Ala.; and W. J. GREENSTREET, Editor of The Mathematical Gazette, Stroud, England.

Let the chord be of length $2b$ in circle of radius a; and take as axes the parallel diameter and the perpendicular bisector of the chord. The distance of the chord from the center is $\sqrt{(a^2-b^2)}$. The required volume is

$$V=2\int_0^b (a^2-x^2)dx-2\pi b(a^2-b^2)=2\pi b(a^2-\tfrac{1}{3}b^2)-2\pi b(a^2-b^2)=\tfrac{4}{3}\pi b^3.$$

This is independent of a, and is equal to the volume of a sphere of diameter $2b$.

Also solved by C. Hornung, J. Scheffer, A. H. Holmes, and the Proposer.